Graduate Aptitude Test in Engineering

Notations:
1. Options shown in green color and with ✔ icon are correct.
2. Options shown in red color and with ✗ icon are incorrect.

Question Paper Name: PH: PHYSICS 1st Feb Shift2
Number of Questions: 65
Total Marks: 100.0

Wrong answer for MCQ will result in negative marks, (-1/3) for 1 mark Questions and (-2/3) for 2 marks Questions.

General Aptitude
Number of Questions: 10
Section Marks: 15.0

Q.1 to Q.5 carry 1 mark each & Q.6 to Q.10 carry 2 marks each.

Question Number : 1  Question Type : MCQ
Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Apparent lifelessness __________________ dormant life.

(A) harbours  (B) leads to  (C) supports  (D) affects

Options:
1. ✔ A
2. ✗ B
3. ✗ C
4. ✗ D

Question Number : 2  Question Type : MCQ
Fill in the blank with the correct idiom/phrase.

That boy from the town was a ____________ in the sleepy village.

(A) dog out of herd  (B) sheep from the heap
(C) fish out of water  (D) bird from the flock

Options:
1. ✗ A
2. ✗ B
3. ✔ C
4. ✗ D

Question Number : 3  Question Type : MCQ
Choose the statement where underlined word is used correctly.

(A) When the teacher eludes to different authors, he is being elusive.
(B) When the thief keeps eluding the police, he is being elusive.
(C) Matters that are difficult to understand, identify or remember are allusive.
(D) Mirages can be allusive, but a better way to express them is illusory.

Options:
1. **A**
2. ✓ B
3. ✓ C
4. ✓ D

Question Number : 4  Question Type : MCQ
Tanya is older than Eric.
Cliff is older than Tanya.
Eric is older than Cliff

If the first two statements are true, then the third statement is:

(A) True
(B) False
(C) Uncertain
(D) Data insufficient

Options:
1. **A**
2. ✓ B
3. ✓ C
4. ✓ D

Question Number : 5  Question Type : MCQ
Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?

(A) 20  
(B) 10  
(C) 8  
(D) 5

Options:
1. **A**
2. ✓ B
3. ✓ C
4. ✓ D

Question Number : 6  Question Type : MCQ
Select the appropriate option in place of underlined part of the sentence.

**Increased productivity necessary reflects greater efforts made by the employees.**

(A) Increase in productivity necessary
(B) Increase productivity is necessary
(C) Increase in productivity necessarily
(D) No improvement required

Options:
1. ✗ A
2. ✗ B
3. ✔ C
4. ✗ D

**Question Number : 7  Question Type : MCQ**

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

**Statements:**
I. No manager is a leader.
II. All leaders are executives.

**Conclusions:**
I. No manager is an executive.
II. No executive is a manager.

(A) Only conclusion I follows.
(B) Only conclusion II follows.
(C) Neither conclusion I nor II follows.
(D) Both conclusions I and II follow.

Options :
1. ✗ A
2. ✗ B
3. ✔ C
4. ✗ D

**Question Number : 8  Question Type : NAT**

In the given figure angle Q is a right angle. PS:QS = 3:1. RT:QT = 5:2 and PU:UR = 1:1. If area of triangle QTS is 20 cm², then the area of triangle PQR in cm² is _______.

![Diagram](image)

**Correct Answer :**

280
Question Number : 9  Question Type : MCQ
Right triangle PQR is to be constructed in the xy-plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P, Q, and R are to be integers that satisfy the inequalities: \(-4 \leq x \leq 5\) and \(6 \leq y \leq 16\). How many different triangles could be constructed with these properties?

(A) 110  (B) 1,100  (C) 9,900  (D) 10,000

Options :
1. ✗ A
2. ✗ B
3. ✓ C
4. ✗ D

Question Number : 10  Question Type : MCQ
A coin is tossed thrice. Let \(X\) be the event that a head occurs in each of the first two tosses. Let \(Y\) be the event that a tail occurs on the third toss. Let \(Z\) be the event that two tails occur in three tosses. Based on the above information, which one of the following statements is TRUE?

(A) \(X\) and \(Y\) are not independent  (B) \(Y\) and \(Z\) are dependent
(C) \(Y\) and \(Z\) are independent  (D) \(X\) and \(Z\) are independent

Options :
1. ✗ A
2. ✓ B
3. ✗ C
4. ✗ D

Physics

Number of Questions: 55
Section Marks: 85.0

Q.11 to Q.35 carry 1 mark each & Q.36 to Q.65 carry 2 marks each.

Question Number : 11  Question Type : MCQ
A satellite is moving in a circular orbit around the Earth. If \(T\), \(V\) and \(E\) are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

(A) \(V = -2T; E = -T\)  (B) \(V = -T; E = 0\)
(C) \(V = -T/2; E = T/2\)  (D) \(V = -3T/2; E = -T/2\)

Options :
1. ✓ A
2. ✗ B
3. ✗ C
4. ** D

Question Number : 12  Question Type : NAT

The Pauli matrices for three spin-$\frac{1}{2}$ particles are $\vec{\sigma}_1$, $\vec{\sigma}_2$, and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is ____________

Correct Answer :  
8

Question Number : 13  Question Type : NAT

The mean kinetic energy of a nucleon in a nucleus of atomic weight $A$ varies as $A^n$, where $n$ is ______________ (upto two decimal places)

Correct Answer : -0.67 to -0.66

Question Number : 14  Question Type : MCQ

Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators, respectively, for a particle. The commutator $[L_x, p_y]$ gives

(A) $-i\hbar p_z$  
(B) 0  
(C) $i\hbar p_x$  
(D) $i\hbar p_z$

Options :
1. ** A
2. ** B
3. ** C
4. ** D

Question Number : 15  Question Type : MCQ
The decay $\mu^+ \rightarrow e^+ + \gamma$ is forbidden, because it violates

(A) momentum and lepton number conservations

(B) baryon and lepton number conservations

(C) angular momentum conservation

(D) lepton number conservation

Options :
1. * A  
2. * B  
3. * C  
4. ✓ D

Question Number : 16 Question Type : MCQ

An operator for a spin-$\frac{1}{2}$ particle is given by $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$, where $\vec{B} = \frac{B}{\sqrt{2}} (\hat{x} + \hat{y})$, $\vec{\sigma}$ denotes Pauli matrices and $\lambda$ is a constant. The eigenvalues of $\hat{A}$ are

(A) $\pm \lambda B/\sqrt{2}$  
(B) $\pm \lambda B$  
(C) $0, \lambda B$  
(D) $0, -\lambda B$

Options :
1. * A  
2. ✓ B  
3. * C  
4. * D

Question Number : 17 Question Type : MCQ

In an inertial frame $S$, two events $A$ and $B$ take place at $(c t_A = 0, \vec{r}_A = 0)$ and $(c t_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame $S'$ moving with a velocity $0.6 c \hat{y}$ with respect to $S$ are given by

(A) $c t'_A = 0; c t'_B = -3/2$  
(B) $c t'_A = 0; c t'_B = 0$

(C) $c t'_A = 0; c t'_B = 3/2$  
(D) $c t'_A = 0; c t'_B = 1/2$

Options :
1. ✓ A  
2. * B  
3. * C  
4. * D
Question Number : 18  Question Type : MCQ

Given that the magnetic flux through the closed loop PQRSP is $\phi$. If $\int_{P}^{R} A \cdot dl = \phi_1$ along PQR, the value of $\int_{P}^{S} A \cdot dl$ along PSR is

(A) $\phi - \phi_1$  (B) $\phi_1 - \phi$  (C) $-\phi_1$  (D) $\phi_1$

Options :
1. ** A
2. □ B
3. ** C
4. ** D

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Question Number : 19  Question Type : MCQ

If $f(x) = e^{-x^2}$ and $g(x) = |x|e^{-x^2}$, then

(A) $f$ and $g$ are differentiable everywhere

(B) $f$ is differentiable everywhere but $g$ is not

(C) $g$ is differentiable everywhere but $f$ is not

(D) $g$ is discontinuous at $x = 0$

Options :
1. ** A
2. □ B
3. ** C
4. ** D

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Question Number : 20  Question Type : MCQ
In Bose-Einstein condensates, the particles

(A) have strong interparticle attraction

(B) condense in real space

(C) have overlapping wavefunctions

(D) have large and positive chemical potential

Options:
1. ✗ A
2. ✗ B
3. ✓ C
4. ✗ D

Question Number : 21  Question Type : MCQ

Consider a system of \( N \) non-interacting spin-\( \frac{1}{2} \) particles, each having a magnetic moment \( \mu \), is in a magnetic field \( \vec{B} = B \hat{z} \). If \( E \) is the total energy of the system, the number of accessible microstates \( \Omega \) is given by

\[
\Omega = \frac{N!}{\frac{1}{2} \left( N - \frac{E}{\mu B} \right)! \cdot \frac{1}{2} \left( N + \frac{E}{\mu B} \right)!}
\]

\[
\Omega = \frac{\left( N - \frac{E}{\mu B} \right)!}{\left( N + \frac{E}{\mu B} \right)!}
\]

\[
\Omega = \frac{1}{2} \left( N - \frac{E}{\mu B} \right)! \cdot \frac{1}{2} \left( N + \frac{E}{\mu B} \right)!
\]

\[
\Omega = \frac{N!}{\left( N + \frac{E}{\mu B} \right)!}
\]

Options:
1. ✓ A
2. ✗ B
3. ✗ C
4. ✗ D

Question Number : 22  Question Type : MCQ
For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

(A) the chemical potential of the photons is zero
(B) photons obey Pauli exclusion principle
(C) photons are spin-1 particles
(D) the entropy of the photons is very large

Options:
1. ✔ A
2. ✗ B
3. ✗ C
4. ✗ D

Question Number : 23  Question Type : MCQ

Consider \( w = f(z) = u(x, y) + iv(x, y) \) to be an analytic function in a domain \( D \). Which one of the following options is NOT correct?

(A) \( u(x, y) \) satisfies Laplace equation in \( D \)
(B) \( v(x, y) \) satisfies Laplace equation in \( D \)
(C) \( \int_{z_1}^{z_2} f(z) \, dz \) is dependent on the choice of the contour between \( z_1 \) and \( z_2 \) in \( D \)
(D) \( f(z) \) can be Taylor expanded in \( D \)

Options:
1. ✗ A
2. ✗ B
3. ✔ C
4. ✗ D

Question Number : 24  Question Type : NAT

The value of \( \int_0^3 t^2 \alpha (3t-6) \, dt \) is _______________ (upto one decimal place)

Correct Answer:
1.3
Question Number : 25  Question Type : MCQ

Which one of the following DOES NOT represent an exclusive OR operation for inputs $A$ and $B$?

(A) $(A + B)\overline{AB}$  
(B) $A\overline{B} + B\overline{A}$  
(C) $(A + B)(\overline{A} + \overline{B})$  
(D) $(A + B)AB$

Options:
1. * A  
2. * B  
3. * C  
4. ✔ D

Question Number : 26  Question Type : MCQ

Consider a complex function $f(z) = \frac{1}{z(z + \frac{1}{2})\cos(\pi z)}$. Which one of the following statements is correct?

(A) $f(z)$ has simple poles at $z = 0$ and $z = -\frac{1}{2}$  
(B) $f(z)$ has a second order pole at $z = -\frac{1}{2}$  
(C) $f(z)$ has infinite number of second order poles  
(D) $f(z)$ has all simple poles

Options:
1. * A  
2. ✔ B  
3. * C  
4. * D

Question Number : 27  Question Type : NAT
In the given circuit, the voltage across the source resistor is 1 V. The drain voltage (in V) is

Correct Answer: 15

Question Number: 28  Question Type: NAT

A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is

Correct Answer: 11

Question Number: 29  Question Type: NAT

A beam of X-ray of intensity \( I_0 \) is incident normally on a metal sheet of thickness 2 mm. The intensity of the transmitted beam is 0.025\( I_0 \). The linear absorption coefficient of the metal sheet (in \( m^{-1} \)) is (upto one decimal place)

Correct Answer: 1844.3 to 1844.5

Question Number: 30  Question Type: NAT
The lattice parameters \(a, b, c\) of an orthorhombic crystal are related by \(a = 2b = 3c\). In units of \(a\), the interplanar separation between the (110) planes is _________ (up to three decimal places).

Correct Answer: 0.445 to 0.450

Question Number: 31 Question Type: MCQ

In a Hall effect experiment, the Hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

(A) \(n \approx p\) \hspace{1cm} (B) \(n > p\) \hspace{1cm} (C) \(\mu_e > \mu_h\) \hspace{1cm} (D) \(m_e^* > m_h^*\)

Options:
1. ** A
2. ** B
3. ✔ C
4. ** D

Question Number: 32 Question Type: MCQ

The space between two plates of a capacitor carrying charges \(+Q\) and \(-Q\) is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct?

![Diagram of capacitor with dielectric materials]

(A) \(\vec{E}\) and \(\vec{D}\) are continuous \hspace{2cm} (B) \(\vec{E}\) is continuous and \(\vec{D}\) is discontinuous

(C) \(\vec{D}\) is continuous and \(\vec{E}\) is discontinuous \hspace{1cm} (D) \(\vec{E}\) and \(\vec{D}\) are discontinuous

Options:
1. ** A
2. ** B
3. ✔ C
Question Number : 33  Question Type : NAT

The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, \( g(E) = CE^n \), where \( C \) is constant. The value of \( n \) is ____________

Correct Answer :
0

Question Number : 34  Question Type : NAT

The dispersion relation for phonons in a one dimensional monatomic Bravais lattice with lattice spacing \( a \) and consisting of ions of masses \( M \) is given by, \( \omega(k) = \sqrt{\frac{2C}{M}} \left[ 1 - \cos(ka) \right] \), where \( \omega \) is the frequency of oscillation, \( k \) is the wavevector and \( C \) is the spring constant. For the long wavelength modes \( (\lambda >> a) \), the ratio of the phase velocity to the group velocity is ____________

Correct Answer :
1

Question Number : 35  Question Type : MCQ

Four forces are given below in Cartesian and spherical polar coordinates.

(i) \( \vec{F}_1 = K \exp(-r^2/R^2) \hat{r} \)
(ii) \( \vec{F}_2 = K(x^2 \hat{y} - y^2 \hat{z}) \)
(iii) \( \vec{F}_3 = K(x^2 \hat{x} + y^2 \hat{y}) \)
(iv) \( \vec{F}_4 = K(\phi \hat{r} / r) \)

where \( K \) is a constant. Identify the correct option.

(A) (iii) and (iv) are conservative but (i) and (ii) are not
(B) (i) and (ii) are conservative but (iii) and (iv) are not
(C) (ii) and (iii) are conservative but (i) and (iv) are not
(D) (i) and (iii) are conservative but (ii) and (iv) are not

Options :
1. ** A
2. ** B
3. ** C
Question Number : 36 Question Type : NAT
Consider a system of eight non-interacting, identical quantum particles of spin-3/2 in a one dimensional box of length \(L\). The minimum excitation energy of the system, in units of \(\frac{\pi^2 h^2}{2mL^2}\) is ____________.

Correct Answer :
5

Question Number : 37 Question Type : NAT

The excitation wavelength of laser in a Raman effect experiment is 546 nm. If the Stokes’ line is observed at 552 nm, then the wavenumber of the anti-Stokes’ line (in \(cm^{-1}\)) is ____________.

Correct Answer :
18513 to 18519

Question Number : 38 Question Type : NAT

The binding energy per molecule of NaCl (lattice parameter is 0.563 nm) is 7.95 eV. The repulsive term of the potential is of the form \(\frac{K}{r^9}\), where \(K\) is a constant. The value of the Madelung constant is ____________ (upto three decimal places)

(Electron charge \(e = -1.6 \times 10^{-19} C\); \(\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}\))

Correct Answer :
1.745 to 1.751

Question Number : 39 Question Type : NAT

Given that the Fermi energy of gold is 5.54 eV, the number density of electrons is ____________ \(\times 10^{28} m^{-3}\) (upto one decimal place)

(Mass of electron = \(9.11 \times 10^{-31} kg\); \(\hbar = 6.626 \times 10^{-34} J.s\); 1 eV = \(1.6 \times 10^{-19} J\))
Correct Answer:
5.9 to 6.0

Question Number : 40  Question Type : NAT

The band gap of an intrinsic semiconductor is $E_g = 0.72 \text{ eV}$ and $m^*_h = 6m^*_e$. At 300 K, the Fermi level with respect to the edge of the valence band (in eV) is at ________ (upto three decimal places)

$k_B = 1.38 \times 10^{-23} J K^{-1}$

Correct Answer :
0.394 to 0.395

Question Number : 41  Question Type : NAT

The number of permitted transitions from $^3\text{P}_{3/2} \rightarrow ^3\text{S}_{1/2}$ in the presence of a weak magnetic field is ________________

Correct Answer :
6

Question Number : 42  Question Type : MCQ
Which one of the following represents the electron occupancy for a superconductor in its normal and superconducting states?

(A)  
(B)  
(C)  
(D)  

Options:
1. ** A
2. ✔️ B
3. ** C
4. ** D

Question Number: 43  Question Type: MCQ
A charge \( -q \) is distributed uniformly over a sphere, with a positive charge \( q \) at its center in (i). Also in (ii), a charge \( -q \) is distributed uniformly over an ellipsoid with a positive charge \( q \) at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?

(A) The dipole moment is zero in both (i) and (ii)
(B) The dipole moment is non-zero in (i) but zero in (ii)
(C) The dipole moment is zero in (i) but non-zero in (ii)
(D) The dipole moment is non-zero in both (i) and (ii)

Options:
1. ✅ A
2. ✗ B
3. ✗ C
4. ✗ D
Consider the circuit shown in the figure, where RC = 1. For an input signal \( V_i \) shown below, choose the correct \( V_o \) from the options:

Options:
1. A
2. B
3. C
4. D

Question Number: 45  Question Type: MCQ
A long solenoid is embedded in a conducting medium and is insulated from the medium. If the current through the solenoid is increased at a constant rate, the induced current in the medium as a function of the radial distance \( r \) from the axis of the solenoid is proportional to

(A) \( r^2 \) inside the solenoid and \( \frac{1}{r} \) outside  
(B) \( r \) inside the solenoid and \( \frac{1}{r^2} \) outside  
(C) \( r^2 \) inside the solenoid and \( \frac{1}{r^2} \) outside  
(D) \( r \) inside the solenoid and \( \frac{1}{r} \) outside

Options:
1. **A**  
2. **B**  
3. **C**  
4. **D**

**Question Number : 46  Question Type : NAT**

In the simple current source shown in the figure, \( Q_1 \) and \( Q_2 \) are identical transistors with current gain \( \beta = 100 \) and \( V_{BE} = 0.7 \, V \)

![Circuit Diagram]

The current \( I_o \) (in mA) is _______ (upto two decimal places)

**Correct Answer :**

5.74 to 5.75

**Question Number : 47  Question Type : MCQ**
Match the phrases in Group I and Group II and identify the correct option.

<table>
<thead>
<tr>
<th>Group I</th>
<th>Group II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P) Electron spin resonance (ESR)</td>
<td>(i) radio frequency</td>
</tr>
<tr>
<td>(Q) Nuclear magnetic resonance (NMR)</td>
<td>(ii) visible range frequency</td>
</tr>
<tr>
<td>(R) Transition between vibrational states of a molecule</td>
<td>(iii) microwave frequency</td>
</tr>
<tr>
<td>(S) Electronic transition</td>
<td>(iv) far-infrared range</td>
</tr>
</tbody>
</table>

Options:
1. ** A  
2. ** B  
3. ** C  
4. ✔ D

Question Number : 48 Question Type : MCQ

Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If \( \vec{F}_c \) and \( \vec{F}_{Co} \) denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

(A) \( \vec{F}_c \) is radially outward and \( \vec{F}_{Co} = \vec{F}_c \)

(B) \( \vec{F}_c \) is radially inward and \( \vec{F}_{Co} = -2\vec{F}_c \)

(C) \( \vec{F}_c \) is radially outward and \( \vec{F}_{Co} = -2\vec{F}_c \)

(D) \( \vec{F}_c \) is radially outward and \( \vec{F}_{Co} = 2\vec{F}_c \)

Options:
1. ** A  
2. ** B  
3. ✔ C  
4. ** D

Question Number : 49 Question Type : NAT

In a rigid-rotator of mass \( M \), if the energy of the first excited state is 1 meV, then the fourth excited state energy (in meV) is _____________
Question Number : 50  Question Type : MCQ

A plane wave \((\hat{x} + i \hat{y})E_0 \exp \left[ i(kz - \omega t) \right]\) after passing through an optical element emerges as \((\hat{x} - i \hat{y})E_0 \exp \left[ i(kz - \omega t) \right]\), where \(k\) and \(\omega\) are the wavevector and the angular frequency, respectively. The optical element is a

(A) quarter wave plate  
(B) half wave plate  
(C) polarizer  
(D) Faraday rotator

Options :
1. ✗ A  
2. ✗ B  
3. ✗ C  
4. ✗ D

Question Number : 51  Question Type : MCQ

The Lagrangian for a particle of mass \(m\) at a position \(\vec{r}\) moving with a velocity \(\vec{v}\) is given by
\[
L = \frac{m}{2} \vec{v}^2 + C \vec{r} \cdot \vec{v} - V(\vec{r}),
\]
where \(V(\vec{r})\) is a potential and \(C\) is a constant. If \(\vec{p}_c\) is the canonical momentum, then its Hamiltonian is given by

(A) \(\frac{1}{2m}(\vec{p}_c + C \vec{r})^2 + V(\vec{r})\)  
(B) \(\frac{1}{2m}(\vec{p}_c - C \vec{r})^2 + V(\vec{r})\)  
(C) \(\frac{\vec{p}_c^2}{2m} + V(\vec{r})\)  
(D) \(\frac{1}{2m} \vec{p}_c^2 + C^2 \vec{r}^2 + V(\vec{r})\)

Options :
1. ✗ A  
2. ✔ B  
3. ✗ C  
4. ✗ D

Question Number : 52  Question Type : MCQ
The Hamiltonian for a system of two particles of masses \( m_1 \) and \( m_2 \) at \( \vec{r}_1 \) and \( \vec{r}_2 \) having velocities \( \vec{v}_1 \) and \( \vec{v}_2 \) is given by
\[
H = \frac{1}{2} m_1 \vec{v}_1^2 + \frac{1}{2} m_2 \vec{v}_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2} \hat{z} \cdot (\vec{r}_1 \times \vec{r}_2),
\]
where \( C \) is a constant. Which one of the following statements is correct?

(A) The total energy and total momentum are conserved
(B) Only the total energy is conserved
(C) The total energy and the \( z \)-component of the total angular momentum are conserved
(D) The total energy and total angular momentum are conserved

Question Number : 53 Question Type : NAT

A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity \( v(0) = 10 \text{ ms}^{-1} \). If the air exerts a frictional force of the form \( f = -kv \), then for \( k = 0.05 \text{ Nm}^{-1} \text{s} \), the velocity (in \( \text{ms}^{-1} \)) at time \( t = 0.2 \text{ s} \) is \( \text{_________} \) (upto two decimal places)

(use \( g = 10 \text{ ms}^{-2} \) and \( e = 2.72 \))

Correct Answer :
4.93 to 4.98

Question Number : 54 Question Type : MCQ

In the nuclear shell model, the potential is modeled as \( V(r) = \frac{1}{2} m \omega^2 r^2 - \lambda \vec{L} \cdot \vec{S}, \quad \lambda > 0 \). The correct spin - parity and isospin assignments for the ground state of \(^{13}C\) is

(A) \( \frac{1^-}{2} : -\frac{1}{2} \)
(B) \( \frac{1^+}{2} : -\frac{1}{2} \)
(C) \( \frac{3^+}{2} : \frac{1}{2} \)
(D) \( \frac{3^-}{2} : -\frac{1}{2} \)

Options :
1. ✔ A
Question Number : 55  Question Type : MCQ

A particle is confined in a box of length $L$ as shown below.

If the potential $V_0$ is treated as a perturbation, including the first order correction, the ground state energy is

(A) $E = \frac{\hbar^2 \pi^2}{2mL^2} + V_0$
(B) $E = \frac{\hbar^2 \pi^2}{2mL^2} - \frac{V_0}{2}$
(C) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{4}$
(D) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{2}$

Options :
1. ** A
2. ** B
3. ** C
4. ✓ D

Question Number : 56  Question Type : NAT

Suppose a linear harmonic oscillator of frequency $\omega$ and mass $m$ is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[ |\psi_0\rangle + e^{\frac{\pi}{2}} |\psi_1\rangle \right]$$

at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle \psi | x | \psi \rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is ___________

Correct Answer :
0

Question Number : 57  Question Type : MCQ
A particle with rest mass $M$ is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the $z$ axis. Their velocities are given by

(A) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ 
(B) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(C) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ 
(D) $\vec{v}_1 = (0.6c)\hat{z}; \quad \vec{v}_2 = (-0.6c)\hat{z}$

Options:
1. **A**
2. ✔️ **B**
3. **C**
4. **D**

Question Number: 58  Question Type: MCQ

In the given circuit, if the open loop gain $A = 10^5$, the feedback configuration and the closed loop gain $A_f$ are

![Circuit Diagram]

(A) series-shunt, $A_f = 9$ 
(B) series-series, $A_f = 10$

(C) series-shunt, $A_f = 10$ 
(D) shunt-shunt, $A_f = 10$

Options:
1. **A**
2. **B**
3. ✔️ **C**
4. **D**

Question Number: 59  Question Type: MCQ
A function \( y(z) \) satisfies the ordinary differential equation \[ y'' + \frac{1}{z} y' - \frac{m^2}{z^2} y = 0, \] where \( m = 0, 1, 2, 3, \ldots \). Consider the four statements P, Q, R, S as given below.

P: \( z^m \) and \( z^{-m} \) are linearly independent solutions for all values of \( m \)
Q: \( z^m \) and \( z^{-m} \) are linearly independent solutions for all values of \( m > 0 \)
R: \( \ln z \) and 1 are linearly independent solutions for \( m = 0 \)
S: \( z^m \) and \( \ln z \) are linearly independent solutions for all values of \( m \)

The correct option for the combination of valid statements is

(A) P, R and S only  (B) P and R only  (C) Q and R only  (D) R and S only

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**Question Number : 60  Question Type : MCQ**

The entropy of a gas containing \( N \) particles enclosed in a volume \( V \) is given by

\[
S = N k_b \ln \left( \frac{a V E^{3/2}}{N^{5/2}} \right),
\]

where \( E \) is the total energy, \( a \) is a constant and \( k_b \) is the Boltzmann constant. The chemical potential \( \mu \) of the system at a temperature \( T \) is given by

(A) \( \mu = -k_b T \left[ \ln \left( \frac{a V E^{3/2}}{N^{5/2}} \right) - \frac{5}{2} \right] \)
(B) \( \mu = -k_b T \left[ \ln \left( \frac{a V E^{3/2}}{N^{5/2}} \right) - \frac{3}{2} \right] \)
(C) \( \mu = -k_b T \left[ \ln \left( \frac{a V E^{5/2}}{N^{3/2}} \right) - \frac{5}{2} \right] \)
(D) \( \mu = -k_b T \left[ \ln \left( \frac{a V E^{3/2}}{N^{3/2}} \right) - \frac{3}{2} \right] \)

Options :
1. ✔ A
2. ☒ B
3. ✗ C
4. ✗ D

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**Question Number : 61  Question Type : NAT**
Let the Hamiltonian for two spin-$\frac{1}{2}$ particles of equal masses $m$, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m} \vec{p}_1^2 + \frac{1}{2m} \vec{p}_2^2 + \frac{1}{2} m \omega^2 (\vec{r}_1^2 + \vec{r}_2^2) + k \vec{\sigma}_1 \cdot \vec{\sigma}_2,$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar \omega = 0.1 eV$ and $k = 0.2 eV$. If the ground state has net spin zero, then the energy (in $eV$) is 

Correct Answer: 
-0.3

Question Number: 62  Question Type: MCQ

The average energy $U$ of a one dimensional quantum oscillator of frequency $\omega$ and in contact with a heat bath at temperature $T$ is given by

\[ (A) \quad U = \frac{1}{2} \hbar \omega \coth \left( \frac{1}{2} \beta \hbar \omega \right) \quad \text{ (B) } U = \frac{1}{2} \hbar \omega \sinh \left( \frac{1}{2} \beta \hbar \omega \right) \]

\[ (C) \quad U = \frac{1}{2} \hbar \omega \tanh \left( \frac{1}{2} \beta \hbar \omega \right) \quad \text{ (D) } U = \frac{1}{2} \hbar \omega \cosh \left( \frac{1}{2} \beta \hbar \omega \right) \]

Options:
1. ✔ A
2. ✗ B
3. ✗ C
4. ✗ D

Question Number: 63  Question Type: NAT

A monochromatic plane wave (wavelength = 600 nm) $E_0 \exp[i(kz - \omega t)]$ is incident normally on a diffraction grating giving rise to a plane wave $E_1 \exp[i(k_1 \cdot \hat{r} - \omega t)]$ in the first order of diffraction. Here $E_1 < E_0$ and $k_1 = |k_1| \left[ \frac{1}{2} \hat{x} + \frac{\sqrt{3}}{2} \hat{z} \right]$. The period (in $\mu m$) of the diffraction grating is ____________ (upto one decimal place)

Correct Answer: 
1.2

Question Number: 64  Question Type: MCQ
The Heaviside function is defined as \( H(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases} \) and its Fourier transform is given by 
\[-2i/\omega.\] The Fourier transform of \( \frac{1}{2}[H(t+1/2) - H(t-1/2)] \) is

(A) \( \frac{\sin \left( \frac{\omega}{2} \right)}{\omega/2} \) 
(B) \( \frac{\cos \left( \frac{\omega}{2} \right)}{\omega/2} \) 
(C) \( \sin \left( \frac{\omega}{2} \right) \) 
(D) 0

Options:
1. ✔ A
2. ✗ B
3. ✗ C
4. ✗ D

Question Number : 65  Question Type : NAT

The atomic masses of \(^{152}_{63}\text{Eu} \), \(^{152}_{62}\text{Sm} \), \(^1\text{H} \) and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the \( Q \)-value of the reaction \(^{152}_{63}\text{Eu} + n \rightarrow ^{152}_{62}\text{Sm} + p \) is \( \text{________} \times 10^{-3} \) amu (upto three decimal places)

Correct Answer :
2.830 to 2.835